Vertical Spin Motion in Circular Ring at Cancellation of g-2 Precession

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1. Perfect Cancellation of g-2 Precession

Vertical spin motion in the circular ring becomes simpler when a g-2 precession is cancelled locally, not on the average. When the cancellation of g-2 precession is perfect, the angle between projections of spin and momentum onto the horizontal plane is invariable. Electric and magnetic fields have some local distortions:

$$\bar{E} = \bar{E}_0 + \Delta\bar{E} \qquad \bar{B} = \bar{B}_0 + \Delta\bar{B}$$

$\bar{E}_0$ and $\bar{B}_0$ – ideal fields without distortions

$$\bar{p} = \bar{p}_0 + \Delta\bar{p} \qquad \bar{p}_0 \text{ – "magic" momentum} \qquad \bar{B}_0 = B_0\bar{e}_z$$

The corrections are relatively small:

$$|\Delta\bar{E}/E_0| \ll 1, \quad |\Delta\bar{B}/B_0| \ll 1, \quad |\Delta\bar{p}/p_0| \ll 1.$$

In this case, there is not any fundamental difference between the twist and saucer distortions. Both distortions are affected by the same field components, B_r and E_z. They correspond to different parts of the same tilted trajectory.

(F.J.M. Farley, Muon EDM Note #24.)

We can take into account the Maxwell equations

$$\oint \bar{E}\cdot d\bar{l} = 0, \quad \oint \bar{B}\cdot d\bar{l} = 4\pi I, \quad d\bar{l} = 2\pi r\bar{e}_\phi d\phi,$$

where I is the current passing through the particle orbit. When $I=0$,

$$\oint E_\phi d\phi = 0, \quad \oint B_\phi d\phi = 0.$$

$$\langle E_\phi \rangle = 0, \quad \langle B_\phi \rangle = 0.$$

Vertical spin displacement:

$$\Delta\xi_z = (\bar{\Omega}\times\bar{\xi})_z \Delta t = (\Omega_r \xi_\phi - \Omega_\phi \xi_r)\cdot\Delta t$$

$\bar{\Omega}$ – angular velocity of spin precession

spin is frozen

$$\langle\Delta\xi_z\rangle = (\langle\Omega_r\rangle \xi_\phi - \langle\Omega_\phi\rangle \xi_r)\cdot\Delta t$$

Bargmann - Michel - Telegdi equation:

$$\frac{d\bar{\xi}}{dt} = \frac{e}{2m}\left\{\left(g-2+\frac{2}{\gamma}\right)(\bar{\xi}\times\bar{B}) - \frac{g-2}{\gamma(\gamma+1)m^2}(\bar{\xi}\times\bar{p})(\bar{p}\cdot\bar{B}) + \frac{1}{\gamma m}\left(g-2+\frac{2}{\gamma+1}\right)\left(\bar{\xi}\times(\bar{E}\times\bar{p})\right)\right\}$$

We neglect the spin precession in the horizontal plane.

In the vertical plane:

$$\bar{\Omega}_\perp = -\frac{e}{2m}\left\{\left(g-2+\frac{2}{\gamma}\right)\bar{B}_\perp - \frac{g-2}{\gamma(\gamma+1)m^2}\bar{p}_\perp(\bar{p}\cdot\bar{B}) + \frac{1}{\gamma m}\left(g-2+\frac{2}{\gamma+1}\right)(\bar{E}\times\bar{p})_\perp\right\}$$

It is possible to neglect products of two or three small quantities. In this approximation

B_ϕ, E_z, p_z are small

$$\bar{p}_\perp(\bar{p}\cdot\bar{B}) = (\lambda p_z B_0 + p_0 B_\phi)p_0\bar{e}_\phi, \quad (\bar{E}\times\bar{p})_\perp = -\lambda(E_z p_0 \bar{e}_r + E_0 p_z \bar{e}_\phi).$$

$$F_z = e\left(E_z + \frac{1}{\gamma m}(\bar{p}\times\bar{B})_z\right) = e\left(E_z - \frac{1}{\gamma m}\lambda B_r p_0\right)$$

$E_r = \lambda E_0$ particle moves CCW $\lambda = 1$ $E_0 > 0$

particle moves CW $\lambda = -1$

The average Lorentz force is zero: $\langle F_z\rangle = 0$.

$$\langle B_r \rangle = \lambda \frac{\gamma m}{p_0} \langle E_z \rangle = \frac{\lambda}{\beta} \langle E_z \rangle .$$

After averaging,

$$\langle p_z \rangle = 0, \quad \langle \bar{p}_\perp (\bar{p} \cdot \bar{B}) \rangle = 0, \quad \langle (\bar{E} \times \bar{p})_\perp \rangle = -\lambda \langle E_z \rangle p_0 \bar{e}_r ,$$

$$\langle \bar{B}_\perp \rangle = \frac{\lambda}{\beta} \langle E_z \rangle \bar{e}_r .$$

Final formula is known:

$$\langle \bar{\Omega}_\perp \rangle = -\frac{e\lambda g}{2\beta\gamma^2 m} \langle E_z \rangle \bar{e}_r \quad \text{or} \quad \langle \Omega_\perp \rangle = \frac{|eg|}{2\beta\gamma^2 m} |\langle E_z \rangle| .$$

The vertical E-field and the field distortions lead to the same effect. Therefore, the systematical error depends only on the average E-field. <u>This is a great advantage of the cancellation of g-2 precession locally, but not on the average.</u>

We can compare the vertical spin motion caused by the vertical electric field and the EDM.

Angular velocity:

$$\bar{\Omega}_{EDM} = -\frac{e\eta}{2m} (\bar{E} + \bar{\beta} \times \bar{B})$$

$$\bar{\Omega}_{EDM} = -\frac{e\eta}{4m} \lambda \beta g x B_0 \bar{e}_r = -\frac{e\eta\lambda g}{4a\gamma^2 m} E_0 \bar{e}_r \qquad x = \frac{1}{1-a(\gamma^2-1)}$$

The effective η factor equals

$$\eta_E = \left| \frac{\Omega_\perp \eta}{\Omega_{EDM}} \right| = \left| \frac{2a\Theta_E}{\beta} \right| \qquad \Theta_E = \frac{\langle E_z \rangle}{E}$$

(nonrelativistic approach)

The effective EDM:

$$\frac{d\bar{\xi}}{dt} = \bar{\mu} \times \bar{B} + \bar{d} \times \bar{E} \qquad \bar{\mu} = \mu\bar{\xi}, \ \bar{d} = d\bar{\xi}, \ \langle \bar{s} \rangle = s\bar{\xi}$$

$$\frac{d\bar{\xi}}{dt} = \frac{\mu}{s}(\bar{\xi} \times \bar{B}) + \frac{d}{s}(\bar{\xi} \times \bar{E})$$

$$\mu = \frac{gse}{2m}, \qquad d = \frac{\eta se}{2m} .$$

For muons ($s = 1/2$) and deuterons ($s = 1$):

$$d^{(\mu)} = \eta \times 4.7 \cdot 10^{-14}\ e \cdot cm$$

$$d^{(d)} = \eta \times 5.3 \cdot 10^{-15}\ e \cdot cm$$

false signal

For 0.5 GeV/c muons

$a = 1.1659 \times 10^{-3}$, $\beta = 0.98$, $d_E = 1.1 \cdot 10^{-16}\ e \cdot cm \times \theta_E$

For 0.4 GeV/c deuterons

$a = -0.143$, $\beta = 0.21$, $d_E = 7.2 \times 10^{-15}\ e \cdot cm \times \theta_E$.

When $\theta_E = 1 \times 10^{-5}$ (10^{-2} mrad)

$d_E = 1.1 \times 10^{-21}\ e \cdot cm$ for muons

$d_E = 7.2 \times 10^{-20}\ e \cdot cm$ for deuterons

The false signal is large. It must be canceled or measured.

2 Imperfect Cancellation of g-2 Precession

For particles with different energies and momenta, the cancellation of g-2 precession cannot be perfect.

More general formula:

$$\bar{\Omega}_\perp = -\frac{e}{2m}\left\{\left(g-2+\frac{2}{\gamma}\right)\bar{B}_\perp - \frac{g-2}{\gamma(\gamma+1)m^2}(\lambda p_z B_0 + p_0 B_\phi)\, p_0 e_\phi - \frac{\lambda}{\gamma m}\left(g-2+\frac{2}{\gamma+1}\right)\left(E_z p_0 \bar{e}_r + E_0 p_z \bar{e}_\phi\right)\right\}$$

The slow g-2 precession causes an appearance of a small radial component of polarization vector.

$$\xi_r = -\Omega_z \xi_\phi^{(0)} t = \pm \Omega_z t, \qquad \xi_\phi \approx \xi_\phi^{(0)} = \pm 1 .$$

If the EDM is taken into consideration,

$$\bar{\Omega}_\perp = -\frac{e}{2m}\left\{\frac{\lambda g}{\beta\gamma^2}\langle E_z\rangle \bar{e}_r + \frac{g}{\gamma} B_\phi \bar{e}_\phi - \frac{\lambda a \beta g \varkappa p_z}{m} B_0 \bar{e}_\phi + \eta\lambda\left(\bar{E}_0 + \beta \bar{B}_0\right)\bar{e}_r\right\}$$

Two causes of a nonzero value of $\langle p_z t\rangle$:

1) coherent betatron oscillation — pitch

$$p_z^{(p)} = p_0 \theta_0^{(pitch)} \cos(\omega_p t + \phi_p) \qquad \text{pitch period}$$

2) field distortions — saucer

$$p_z^{(s)} \sim p_0 \Theta, \quad p_z^{(s)}(t + nT_c) = p_z^{(s)}(t), \quad n = 1, 2, 3, \ldots$$

cyclotron period

more complicated dependence

The evaluations:

$$\Delta\xi_z^{(e)} = -\frac{eg}{2\gamma m}\Omega_z \xi_\phi^{(0)} \int_0^t B_\phi t' dt' \sim \Omega_z \theta t, \qquad \theta = \frac{\langle |B_\phi| \rangle}{B_0}$$

$$\Delta\xi_z^{(p)} \sim \frac{a g \varkappa (\gamma^2 - 1)\omega_c \Omega_z}{\omega_p} \theta_0^{(pitch)} t,$$

$$\Delta\xi_z^{(s)} \sim a\varkappa(\gamma^2 - 1)\Omega_z \Theta t, \qquad \Theta \Rightarrow \text{saucer distortion angle}$$

False signals are of orders

$$\eta_p \sim \frac{a\beta\gamma\Omega_z\theta_0^{(pitch)}}{\omega_p}, \quad \eta_s \sim \frac{a\beta\gamma\Omega_z\Theta}{\omega_c}, \quad \eta_e \sim \frac{\Omega_z\theta}{\beta\gamma\omega_c}.$$

For 0.5 GeV/c muons, they correspond to the errors of measurement of EDM

$$d_p \sim 10^{-16}\, e\cdot cm \times \frac{\Omega_z\theta_0^{(pitch)}}{\omega_p}, \quad d_s \sim 10^{-16}\, e\cdot cm \times \frac{\Omega_z\Theta}{\omega_c},$$

$$d_e \sim 10^{-14}\, e\cdot cm \times \frac{\Omega_z\theta}{\omega_c}$$

For 0.4 GeV/c deuterons

$$d_p \sim 10^{-16}\, e\cdot cm \times \frac{\Omega_z\theta_0^{(pitch)}}{\omega_p}, \quad d_s \sim 10^{-16}\, e\cdot cm \times \frac{\Omega_z\Theta}{\omega_c},$$

$$d_e \sim 10^{-14}\, e\cdot cm \times \frac{\Omega_z\theta}{\omega_c}$$

Usually the pitch angle is relatively large

$$\theta_0^{(pitch)} \gg \Theta, \quad \theta_0^{(pitch)} \gg \theta$$

Ω_z is small as compared with the angular velocity of g-2 precession:

$$\Omega_z \ll \Omega_{g-2} = a\gamma\omega_c$$

When $\Omega_z \sim 10^{-2} \times \Omega_{g-2} \sim 10^{-2} \times a\gamma\omega_c$, $\theta_0^{(pitch)} = 1\times 10^{-3}$, $\omega_p \approx 4\omega_c$,

the false signal is of order of

10^{-24} e·cm for muons

10^{-23} e·cm for deuterons

Summing over all detectors makes it possible to decrease false signal considerably.

3 Vertical Spin Motion for Particles Polarized Radially

Polarization vector is collinear to the radial axis
— the EDM signal vanishes
— the false signal due to vertical E-field vanishes
— the false signal due to radial B-field vanishes

Therefore, the beam polarized radially can be used in order to calibrate detectors. However, an imperfect cancellation of g-2 precession causes a false signal that is comparatively great.

Initial particle polarization is radial. Precession with an angular velocity Ω_z leads to an appearance of longitudinal particle polarization.

$$\xi_r = \xi_r^{(0)} = \pm 1 \,, \qquad \xi_\phi = \Omega_z \xi_r^{(0)} t = \pm \Omega_z t \,.$$

Vertical spin motion:

$$\Delta \xi_z = \Omega_z \xi_r^{(0)} \int_0^t \Omega_r(t') t' dt' - \langle \Omega_\phi \rangle \xi_r^{(0)} t \,.$$

After averaging, $\langle \Omega_\phi \rangle = 0$.

The integral is determined by the effective value of B_r, that is the sum of three summands:

$$B_r = \langle B_r \rangle + B_r^{(p)} + B_r^{(s)}$$

The quantity $B_r^{(p)}$ is determined by the pitch and have the sinusoidal dependence.

The quantity $B^{(s)}$ is determined by the saucer distortion. Its period equals T_c.

As a result,

$$\Omega_r = -\frac{e\lambda q}{2\beta\gamma^2 m}\langle E_z\rangle - \frac{e}{2m}\left(g-2+\frac{2}{\gamma}\right)\left[\frac{B_0\theta_0^{(pitch)}\omega_p}{\omega_c}\cos(\omega_p t+\phi_p)+B_r^{(s)}(t)\right]$$

The vertical coherent betatron oscillation is a principal source of systematical errors, although both the vertical E-field and the saucer distortion can cause systematical errors of the same order.

Vertical spin displacement

$$\Delta\xi_z \sim \Omega_z\theta_0^{(pitch)}t$$

False signal

$$\eta \sim \frac{\Omega_z\theta_0^{(pitch)}}{\beta\gamma\omega_c}$$

For 0.5 GeV/c muons $d \sim 10^{-14}\,e\cdot cm \times \frac{\Omega_z\theta_0^{(pitch)}}{\omega_c}$

For 0.4 GeV/c deuterons $d \sim 10^{-4}\,e\cdot cm \times \frac{\Omega_z\theta_0^{(pitch)}}{\omega_c}$

When $\Omega_z \sim 10^{-2}\times\Omega_{g-2} \sim 10^{-2}\times a\gamma\omega_c$, $\theta_0^{(pitch)} = 1\times 10^{-3}$,

the false signal is of order of

$\sim 10^{-21}$ e·cm for muons

$\sim 10^{-20}$ e·cm for deuterons

These values are too large.

i) Spin precession in the horizontal plane can be strongly restricted.

ii) There is the good way to detect this systematical error. The error can be increased at least 10 times by increasing the value of Ω_z. Such a false signal can be measured. Since the dependence of a vertical spin displacement on Ω_z is linear, the systematical error corresponding to smaller Ω_z can be determined.

Therefore, use of beam polarized radially to calibrate detectors is possible.

4 Discussion

The corrections caused by the imperfect cancellation of g-2 precession are comparatively small. The false signal is affected by the average vertical electric field. We suppose that the main magnetic field, B_0, depends on a kind of particles, and the main electric field, E_0 is fixed.

When we search for the EDM, the ring with larger radius and larger B_0 is better.

When we measure the average vertical electric field, the measurement is performed with an another particle beam. The ring is the same. The background should be maximum. For particles,

$$g = \frac{2\mu m}{es}.$$

For nuclei, the following relation is more convenient:

$$g = \frac{2\mu_{rel}\mu_N m}{Zes} = \frac{\mu_{rel} m}{Zsm_p} \approx \frac{\mu_{rel} A}{Zs},$$

μ_N is the nuclear magneton

μ_{rel} is a magnetic moment in nuclear magnetons

Z is the atomic number

A is the mass number

When $r = 7.11\,m$, $E_0 = 2\,MV/m$, we obtain:

i) for muons $\beta = 0.98$, $\gamma = 4.94$, $\Omega_\perp = 2.38 \times 10^5\,rad/s \times \theta_E$;
ii) for deuterons $\beta = 0.206$, $\gamma = 1.02$, $\Omega_\perp = -12.7 \times 10^5\,rad/s \times \theta_E$;
iii) for protons $\beta = 0.151$, $\gamma = 1.01$, $\Omega_\perp = 115 \times 10^5\,rad/s \times \theta_E$;
iv) for ^6Li $\beta = 0.180$, $\gamma = 1.02$, $\Omega_\perp = -14.1 \times 10^5\,rad/s \times \theta_E$.

The systematical error is
5.3 greater for the deuteron than for the muon;
48.6 times greater for the proton than for the muon;
9.1 times greater for the proton than for the deuteron.

In the considered conditions the proton momentum is $p_0 = 0.14\,GeV/c$, the magnetic field is $B_0 = 0.023\,T$, the ratio of electric and magnetic forces is $E_0/(\beta B_0) = 1.9$.

Due to the small proton momentum, CBO are less.

5 Conclusions

1. When the g-2 precession is cancelled locally, all the experimental errors caused by the spin motion become similar to the well-known error caused by the vertical electric field.

2. This error is large. In order to measure it, the proton beam can be used.

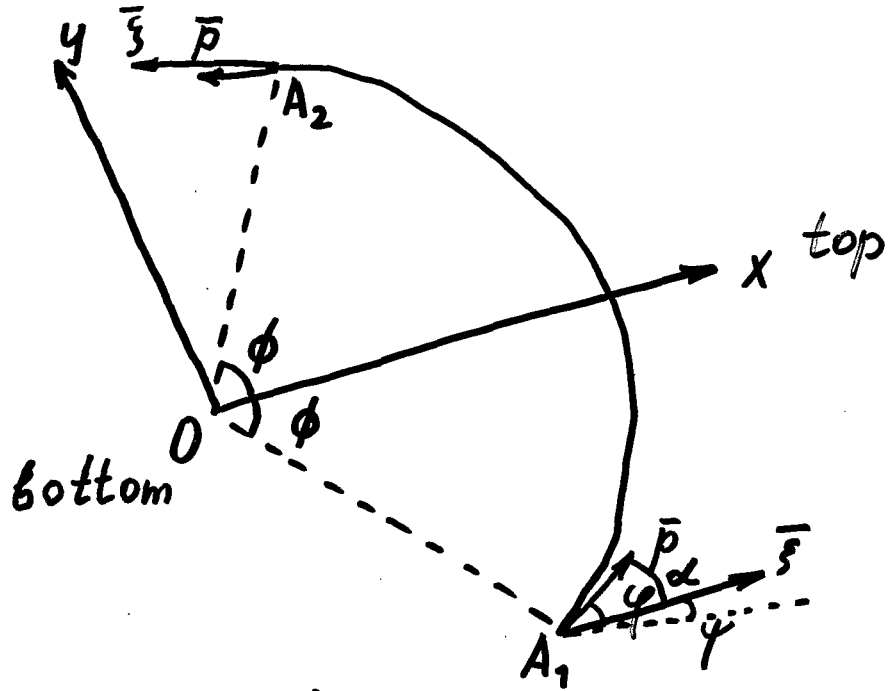
3. The imperfect cancellation of g-2 precession gives a systematical error that is considerably less.

Vertical Spin Motion in a Ring with Non-local g-2 Cancellation

The electric and magnetic fields can be noncontinious. If there is several separated modules of electric and magnetic fields, each module occupies a certain circular sector. When a field direction is deflected from the vertical, we get the field distortion.

Let the B-field region extends on the angle 2ϕ. The axis x and y are parallel and orthogonal to the circular sector bisectrix, respectively. The initial angle between the momentum direction and the axis x equals $\varphi = \frac{\pi}{2} - \phi$. The initial angle between the spin direction and the axis x, ψ, is arbitrary. In this case the angle between the momentum and spin directions, α, is also arbitrary.

$$\alpha = \psi - \varphi = \psi + \phi - \frac{\pi}{2}$$



Let us consider the saucer corrections.

Θ - saucer distortion angle

For the particle flying into the magnetic field,

$$p_x^{(1)} = p\cos\varphi, \quad p_y^{(1)} = p\sin\varphi, \quad p_z^{(1)} = p\Theta\cos\varphi,$$

$$\xi_x^{(1)} = \cos\psi, \quad \xi_y^{(1)} = \sin\psi, \quad \xi_z^{(1)} = \Theta\cos\psi, \qquad \text{point } A_1$$

For the particle flying out from the magnetic field,

$$p_x^{(2)} = -p\cos\varphi, \quad p_y^{(2)} = p\sin\varphi, \quad p_z^{(2)} = -p\Theta\cos\varphi, \qquad \text{point } A_2$$

$$\xi_x^{(2)} = \cos(\psi+\varphi_m), \quad \xi_y^{(2)} = \sin(\psi+\varphi_m), \quad \xi_z^{(2)} = \Theta\cos(\psi+\varphi_m),$$

$$\varphi_m = 2(1+a\gamma)\varphi.$$

The vertical spin displacement in the magnetic field equals:

$$\Delta\xi_z = \xi_z^{(2)} - \xi_z^{(1)} = -2\Theta\sin(\phi(1+a\gamma))\cos(\alpha+\phi a\gamma).$$

In the points A_1 and A_2 the vertical projection of the particle momentum changes quickly at a small distance (a kick occurs).

$$\Delta\xi_z^{(m)} = (1+a\gamma)\xi_p \cdot \frac{\Delta p_z}{p}$$

At the point A_1 $\quad \Delta\xi_z^{(1)} = p\,\Theta\cos\varphi\cos\alpha(1+a\gamma)/p = (1+a\gamma)\Theta\cos\varphi\cos\alpha$

At the point A_2 $\quad \Delta\xi_z^{(2)} = (1+a\gamma)\Theta\cos\varphi\cos(\alpha+2\phi a\gamma)$

The summary kick correction equals

$$\Delta\xi_z^{(k)} = 2(1+a\gamma)\Theta\sin\phi\cos(\alpha+\phi a\gamma)\cos(\phi a\gamma)$$

The formula for the vertical spin displacement per particle revolution:

$$\Delta\xi_z^{(sum)} = 2\Theta\cos(\alpha+\phi a\gamma)\left[a\gamma\sin\phi\cos(\phi a\gamma) - \cos\phi\sin(\phi a\gamma)\right].$$

Approximately,

$$\Delta\xi_z^{(sum)} = 2\Theta a\gamma\cos(\alpha+\phi a\gamma)(\sin\phi - \phi\cos\phi).$$

When ϕ is small, $\Delta\xi_z^{(sum)} \sim \phi^2$.

An analogous dependence takes place for the twist distortion.

$$\Delta\xi_z = 2\theta \sin(\phi(1+a\gamma)) \sin(\alpha+\phi a\gamma)$$

Approximately,

$$\Delta\xi_z = 2\theta \sin\phi \sin(\alpha+\phi a\gamma)$$

θ is the twist distortion angle

Therefore, non-local cancellation of g-2 precession give additional systematical errors. In all probability, these errors cannot be corrected with another particles. Local cancellation of g-2 precession is more preferable.