12 Nov. 03
EDM Collaboration Meeting
BNL

Design for a longer spin coherence time

Yuri F. Orlov, Cornell U.

a typical lattice

B+E are combined: $\omega_a \equiv 0$, not
$\langle \omega_a \rangle = 0$.

$\langle R \rangle = 30\,m$ is fixed.

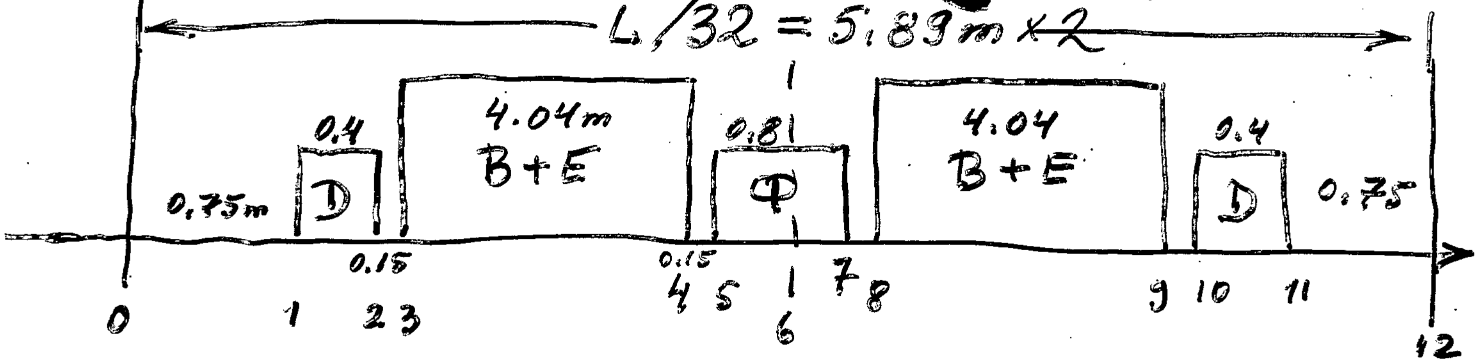
Biggest free intervals, $\ell = 1.5\,m$
are fixed.

Intervals between (B+E)'s and quadrupoles etc. are fixed, 0.15 m

Not so short (B+E)'s.

Not so few intervals, some 16 (8 of them for polarimeters) seem OK

Therefore, the ring:



$$\boxed{\text{Everything is defined by } E_R}$$

$$\begin{aligned}
&0.15 \times 4 \times 16 = 9.6\,m \\
+\;&1.5 \times 16 = 24\,m \\
+\;&0.4 \times 4 \times 16 = 25.6\,m \\
\hline
&59.2\,m
\end{aligned}$$

$$L = 2\pi \times 30 = 188.5\,m$$

Therefore, $L_\mu = L - 59.2 = 129.3$

$$\boxed{R_\mu = L_\mu / 2\pi = 20.6\,m}$$

$$\left.\begin{aligned} p &= 0.3\left(B - \frac{E}{\beta}\right) R_\mu \\ B &= \frac{1}{|a|}\left[|a| + \left(\frac{m}{p}\right)^2\right]\beta E_R \end{aligned}\right\} \quad p = 0.3\, E_R R_\mu \left[\frac{|a| + \left(\frac{m}{p}\right)^2}{|a|}\beta - \frac{1}{\beta}\right]$$

Therefore,

$$E_R = \frac{1}{0.3 R_\mu} \frac{p|a|\beta}{\beta^2\left[|a| + \left(\frac{m}{p}\right)^2\right] - |a|}$$

R_μ in m
E_R in T
p in GeV/c

$$m = 1.8756 \text{ GeV}$$

$$a = -0.14301$$

$$\beta = \frac{p/m}{\sqrt{(p/m)^2 + 1}}, \quad \gamma = \sqrt{(p/m)^2 + 1}$$

For $E_R = 3.5$ MeV/m $= 1.167\times10^{-2}$ T

$p = 0.859$ GeV/c ; $K = 187$ MeV

$B = 0.16713$ T

$\gamma = 1.09975$; $\beta = 0.416154$

$\langle R \rangle = 30\,m$; $R_M = 20.6\,m$

Since $E_R = \dfrac{E_0}{\left(1+\frac{x}{R}\right)}$ (for $y=0$),

I chose $B = \dfrac{B_0}{\left(1+\frac{x}{R}\right)}$.

$\omega_a \propto B + kE$. For a given k,

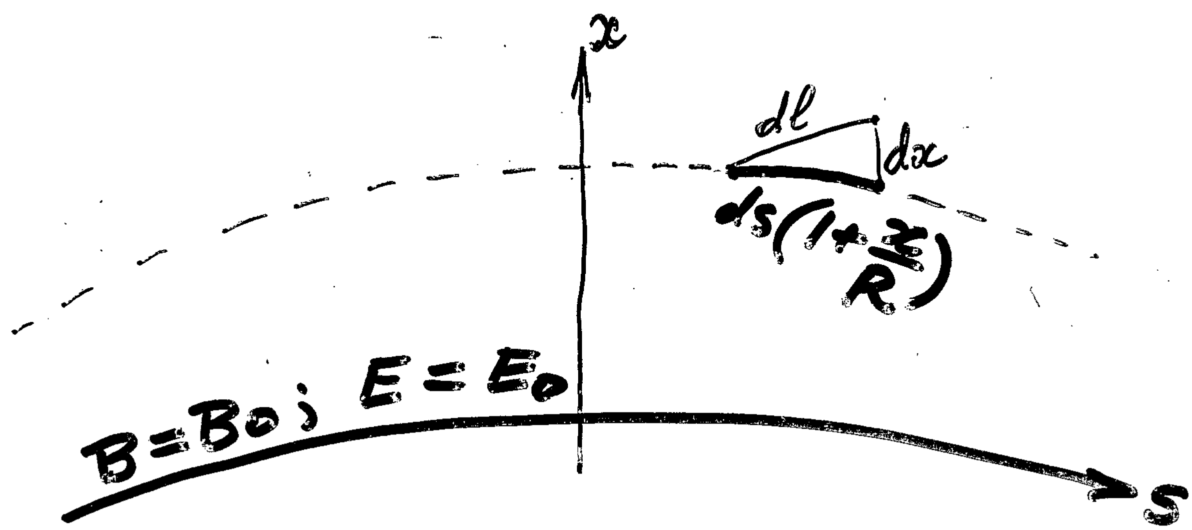
if $\omega_a = 0$ at the central orbit,

then $\omega_a = 0$ (in plane $y=0$) along

<u>any</u> element of the particle

trajectory, inside $(B+E)$.

Integrating of fields along real trajectories inside B+E.



$k = \left(|a| + \left(\frac{m}{p}\right)^2\right)\beta/|a|$

$$\int (B+kE)\,d\ell = \int \frac{B_0+kE_0}{\left(1+\frac{x}{R}\right)} \sqrt{ds^2\left(1+\frac{x}{R}\right)^2 + dx^2} =$$

$$\cong \int (B_0+kE_0)\,ds\left(1+\frac{1}{2}\vartheta_x^2\right)$$

Thus, if $B_0+kE_0 \propto \omega_a = 0$, then

$$\int (B+kE)\,d\ell = 0$$

Inside BE, with respect to $\frac{\Delta p}{p}$, only k is relevant.

When momentum p is given, betatron x-oscillations (in the plane $y=0$) also do not change the balance of B and E fields and the ω_a-cancellation inside the B+E sections.

(Nanotheless, betatron oscillations are one of two main ~~soarses~~ of decoherence, but for different reasons I will explain later. Another source is $\Delta p/p$.)

Thus, inside (B+E) sections, with

$$B = \frac{B_0}{\left(1+\frac{x}{R}\right)},$$

only the coefficient

$$\left(|a| + \left(\frac{m}{p}\right)^2\right)\beta$$

is responsible for the decoherence due to $\frac{\Delta p}{p}$. (Lenses etc. outside BE's will be considered separately later.)

Calculations:

inside B+E,

if at the ideal orbits

$$\omega_a = \frac{e}{m}\left\{ |a| B_0 - \left(|a| + \underbrace{\left(\frac{m}{p_0}\right)^2}_{\gamma_0} \right) \beta_0 E_{R0} \right\} = 0, \quad \text{then}$$

at $p = p_0 + \Delta p$

$$\boxed{\Delta\omega_a = -\frac{e}{m}\left\{ \frac{1}{\gamma_0^2}\left(|a| B_0 - \frac{2E_{R0}}{\beta_0}\right)\frac{\Delta p}{p} + \left[\frac{E_{R0}}{\beta_0 \gamma_0^4} - \frac{3}{2}\frac{\beta_0^2}{\gamma_0^2}\left(|a| B_0 - \frac{2E_{R0}}{\beta_0}\right)\right]\left(\frac{\Delta p}{p}\right)^2 \right\}}$$

(I use $\left(\frac{m}{p}\right)^2 = \left(\frac{m}{p_0}\right)^2 \left[1 - 2\frac{\Delta p}{p} + 3\left(\frac{\Delta p}{p}\right)^2\right]$,

$$\frac{\Delta\beta}{\beta} = \frac{1}{\gamma^2}\frac{\Delta p}{p} - \frac{3}{2}\frac{\beta^2}{\gamma^2}\left(\frac{\Delta p}{p}\right)^2,$$

etc)

For my lattice,

$$\boxed{\Delta\omega_a = \frac{e}{m}|a| B_0 \left(1.1133\frac{\Delta p}{p} - 1.0913\left(\frac{\Delta p}{p}\right)^2\right)}$$

(only inside (B+E))

Can we compensate these terms by choosing properly the quadrupoles, sextupoles, etc?

We cannot compensate the linear $\Delta p/p$ term, in principle. Only the averaging $\Delta p/p$, $\langle \Delta p/p \rangle = 0$, using RF cavities, i.e., synchrotron stability, can help.

The reason of the trouble:

$$pc = e(B - E/\beta)R$$

$$\frac{\Delta R}{R} \cong \frac{1}{\gamma_x^2}\frac{\Delta p}{p} \approx \frac{1}{16}\frac{\Delta p}{p} \ll \frac{\Delta p}{p},$$

so $R \approx$ const.

$E \ll B$.

Hence, $\frac{\Delta B}{B} \approx + \frac{\Delta p}{p}$, whatever we do.

But $\left(\frac{\Delta p}{p}\right)^2$ can be compensated by sextupole fields!

In general, when $\frac{\Delta p}{p} \neq 0$,

$$\boxed{X_{eq}(s) \equiv \Delta R(s) = D(s)\,\frac{\Delta p}{p} + d(s)\left(\frac{\Delta p}{p}\right)^2 + \ldots}$$

Equations:

Inside B+E (our lattice):

$$\frac{d^2 D}{ds^2} = \frac{1}{R_M}$$

$$\frac{d^2 d(s)}{ds^2} = \frac{1}{R_M}\left(\frac{D(s)}{R_M} - 1\right) + \frac{\beta}{R_M}\left(\frac{E_0}{B_0}\right)\left(\frac{dD}{ds}\right)^2$$

Outside B+E:

- put 3'' together with B'. This must be changed

$$\frac{d^2 D(s)}{ds^2} - \frac{n}{R_M^2} D = 0$$

$$\frac{d^2 d(s)}{ds^2} - \frac{n}{R_M^2} d(s) = -\frac{n}{R_M^2} D(s) - \frac{1}{2}\frac{B'' D^2}{BR}$$

$n_1 = 163$; $B_1' \equiv \frac{\partial B_1}{\partial x} = -n_1 \frac{B}{R_M} = 1.32\ T/m$; $B = 0.1671\ T$

$n_6 = 145$; $B_6' = -n_6 \frac{B}{R_M} = 1.18\ T/m$ $\quad R_M = 20.6\ m$

Results of calculations

$$\Delta\omega_{linear} = \frac{e}{m}|a|B_0 \left\{ \underbrace{\frac{l_1}{L/32}\left(|n_6|\frac{D_6}{R_M} - n_1\frac{D_1}{R_M}\right)}_{0.7\,\Delta p/p} + \underbrace{\frac{l_M}{L/32}\frac{1}{\gamma^2}\left(1-\frac{2E_0}{|a|\beta B_0}\right)}_{0.7\frac{\Delta p}{p}} \right\}\frac{\Delta p}{p}$$

$$\cong \frac{e}{m}|a|B_0\, 1.4\,\frac{\Delta p}{p} \qquad 0.7 = \frac{l_M}{L/32} \quad !$$

$$L/32 = 5.89\,m$$

$$\Delta\omega_{nonlin} = \frac{e}{m}|a|B_0\left\{\frac{l_1}{L/32}\left[\frac{1}{2}\frac{B_1'' D_1^2 + B_6'' D_6^2}{B_0} - n_1 d_1 + |n_6| d_6\right] - \right.$$

ΔB outside BE

Coefficient k inside BE

$$- \frac{l_M}{L/32}\left[\frac{E_0}{B_0\beta|a|\gamma^4} + \frac{3}{2}\frac{\beta^2}{\gamma^2}\left(\frac{2E_0}{B_0\beta|a|} - 1\right)\right] +$$

From synchrotron equilibrium condition

$$\left. + \underbrace{\frac{l_M}{L/32}\left[\frac{2E_0}{B_0\beta|a|} - 1\right]\frac{\frac{3}{2}\beta^2/\gamma^2 + \alpha_1}{1-\gamma^2\alpha}}\right\}\left(\frac{\Delta p}{p}\right)^2$$

Obviously, we need $B'' > 0$,

$$\frac{1}{2}\frac{B'' x^2}{B} \sim 10^{-3} \text{ at } x \sim 3\,\text{cm}$$

Nontrivial appearance of the last term

The influence of the synchrotron stability

$$\langle \tau \rangle = \frac{2\pi}{\omega_{RF}} = \text{constant}$$

For a particle,

$$\tau = \frac{L}{v} = \frac{L_0 + \Delta L}{v_0 + \Delta v} = \tau_0 \left\{ \left(\left\langle \frac{D}{R} \right\rangle - \frac{1}{\gamma^2} \right) \frac{\Delta p}{p} + \right.$$

$$\left. + \left[\left\langle \frac{d}{R} \right\rangle + \frac{3}{2} \frac{\beta^2}{\gamma^2} \right] \left(\frac{\Delta p}{p} \right)^2 \right\}$$

$\Rightarrow$ a shift of the equilibrium momentum p:

$$\left(\frac{\Delta p}{p} \right)_{eq} = 0 \Rightarrow \left\langle \frac{\Delta p}{p} \right\rangle = \frac{\left\langle \frac{d}{R} \right\rangle + \frac{3}{2} \frac{\beta^2}{\gamma^2}}{1/\gamma^2 - \left\langle \frac{D}{R} \right\rangle} \overline{\left(\frac{\Delta p}{p} \right)^2}$$

Therefore, now, the $\Delta\omega_{linear}$ above oscillates not about $\frac{\Delta p}{p} = 0$, but about $\left\langle \frac{\Delta p}{p} \right\rangle \neq 0$, $\left\langle \frac{\Delta p}{p} \right\rangle \sim \left(\frac{\Delta p}{p} \right)^2$.

In $\Delta\omega_{linear}$ now $\frac{\Delta p}{p} \to \left(\frac{\Delta p}{p} - \left\langle \frac{\Delta p}{p} \right\rangle \right)$.

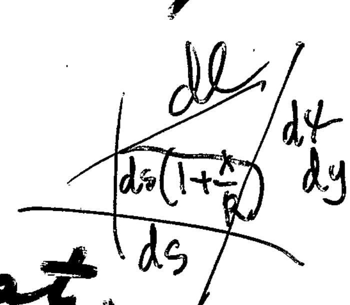
Thus, the $\Delta\omega_{linear}$ is averaged due to synchrotron oscillations, and $\Delta\omega_{nonlin}$ is compensated by the sextupole quadratic B-fields.

What about betatron oscillations?

The problem is that betatron osc's change the actual length of the particle orbit. And since $\tau = \frac{L}{v}$ is fixed by RF, v, and hence, p is shifted (individually, differently for diffent oscillations)

$$\frac{\Delta L_0}{L} = \frac{L_0}{L}\int dl = \frac{1}{L}\int\sqrt{\left(1+\frac{x}{R}\right)^2 ds^2 + dx^2 + dy^2} =$$

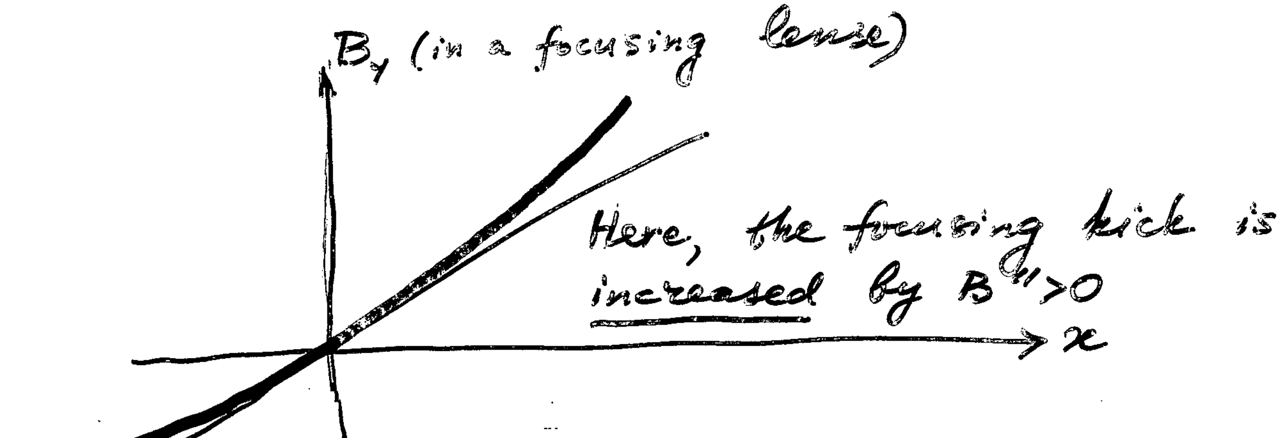
$$\cong \frac{\cancel{L}}{\cancel{L}}\int ds\left[1 + \frac{x}{R} + \frac{1}{2}\vartheta_x^2 + \frac{1}{2}\vartheta_y^2\right]$$



If our beam will be flat, $\langle\vartheta_y^2\rangle \to 0$, then we can cure the rest using again B''.

This part of the work is in progress. For our two problems — two parameters, B_1'' and B_5''. The signs, $B''>0$, are correct!

The idea is to shift the individual particle orbit, $R \to (R - \Delta R)$ with the help of $B''>0$, $\Delta R \propto \langle x^2 \rangle / R$



As a result, the particle is kicked unequally, into inside the ring, and got $\Delta R < 0$.

This may compensate $\Delta R \sim x^2/R > 0$.