EDM collaboration
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Twist & Saucer effects

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"Twist" was formally analysed in the EDM Note #26, 12/05/02, and then was explained also at the EDM collaboration, BNL, 7-8 Feb. 2003.

It is called "a Berry phase effect" in our article sent to PRL.

According to formula (22) of Note #26,

$$\left(\eta_d\right)_{false} = \left(\frac{\delta\omega_{a0}}{\omega_c}\right)\frac{\sin\phi}{k\beta\gamma}\vartheta_L\,,\quad \vartheta_L = \frac{(1+a)B_{L0}}{B},$$

$$B_L = B_{L0}\cos k\omega_c t$$

$$\delta\omega_a = \delta\omega_{a0}\cos(k\omega_c t - \phi).$$

That is, my $\sin\phi$ has a meaning <u>completely different</u> from that of A. Silenko.

With respect to spin/beam dynamics, it is clear that the generalized cylindrical coordinates x, y, s are the most adequate. The only problem is: what are centrifugal forces for spins?
⇒ a pure formal approach;

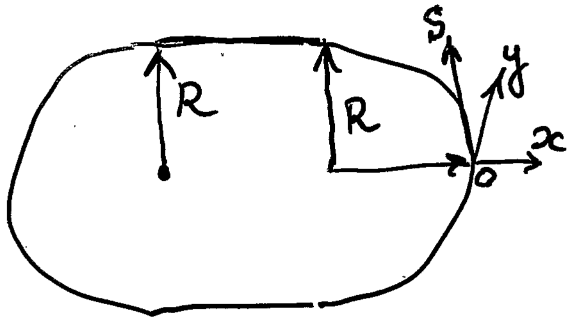
In addition, sometimes, the matrix approach is the most precise (since the spin eq's are linear).

I do not recommend changes of reference planes, surfaces, lines, even coordinates, for a "better" analysis of the needed relations between vectors, tensors etc., and violations of these relations.

We must remember also that particles "know" only fields. Therefore, any "twist" must be translated into field with the help of a defined fixed language.

My approach to such things as system of coordinates, reference plane, etc (intensively discussed in my lab. in 1960th)

Our reference plane (or any other relevant surface) is assumed to be defined by geodesical work.
The closed curve ~~at~~ on that surface, chosen as a coordinate "exis" is also defined <u>geodesicaly</u>. It can or cannot be a future actual closed orbit
For example, this closed curve



is very convenient as a basis for a coordinate system, but <u>cannot</u> be a real closed orbit.
Then, whatever happen with particles, fields, and lattice elements, this system of coordinates remain usually unchanged. This way, the rules and the language are defined unambiguously.

Twist

The foundation of the ring: mutually separate, independent sub-foundations for every heavy element, like a (B+E).

We have, say, $N=32$ such sub-foundations.

Let $\vartheta(n)$, $n=1,2,\dots,32$, be some parameter.

$$\vartheta = \vartheta(n) = a_0 + \sum_{k=1}^{N} a_k e^{i2\pi kn}$$

$$a_k N = \sum_{n=1}^{N} \vartheta(n) e^{-i2\pi kn}$$

$$|a_k|^2 N^2 = N\,\overline{\vartheta^2}, \quad a_0 = \frac{1}{N}\sum_n \vartheta(n)$$

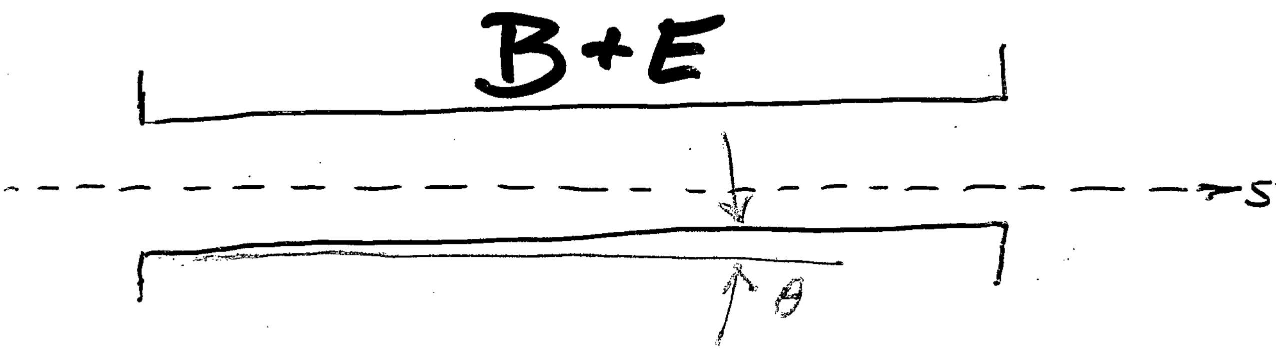
$$\boxed{a_k = |a_k| e^{i\phi_k} = \frac{\sqrt{\overline{\vartheta^2}}}{\sqrt{N}} e^{i\phi_k}}$$

Let $\vartheta(n)$, $n=1,2,\dots,32$, are rotation angles around radial axes.

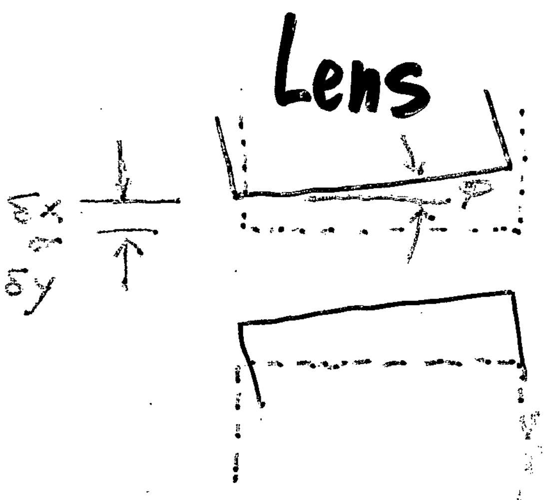
Then $\vartheta(n) = B_L(n)/B_0$

$$\sum_n B_L(n) = 0$$

$$\langle B_L(s)\,\delta\omega_a(s)\rangle \neq 0$$



$$B_L = \theta B_v$$



$$B_L = \frac{\partial B_v}{\partial x}\,\delta x \cdot \phi$$

Plus in fringe fields

Let there be also independent perturbations (local violations) of the (g-2) cancellation:

$$\delta\omega_a = \delta\omega_a(n) = \langle\delta\omega_a\rangle + \sum_k (\delta\omega_a)_k e^{i2\pi kn}$$

$$(\delta\omega_a)_k = |\delta\omega_{ak}| e^{i\psi_k} = \frac{1}{N}\sqrt{\overline{(\delta\omega_a)^2}}$$

Resonances between the same harmonics of $\delta\omega_a$ and B_L are analysed in my Note 26.

For a given k, the expectation values are

$$\omega_a(t) = \delta\omega_a \cos(2\pi kt/T) \qquad \text{(formula (6) of \#26)}$$

$$\delta\omega_a = \frac{1}{\sqrt{N}}\sqrt{\overline{\delta\omega_a^2}}$$

$$\vartheta(t) \equiv B_L(t)/B_0 = \frac{b}{B_0}\cos(2\pi kt/T+\phi) \qquad \text{((7) of \#26)}$$

$$b/B_0 = \frac{1}{\sqrt{N}}\sqrt{\overline{\vartheta^2}}$$

(in "smooth" approximation)

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$$s'_L = s_L / \gamma . \tag{9}$$

s denotes the lab-, and s' the rest-frame spin. All this leads to the following spin equations:

$$ds'_L / dt = s'_R \delta\omega_a \cos 2\pi kt / T , \tag{10}$$

$$ds'_R / dt = -s'_L \delta\omega_a \cos 2\pi kt / T + s'_V \omega_L \cos(2\pi kt / T + \phi) , \tag{11}$$

$$ds'_V / dt = -s'_R \omega_L \cos(2\pi kt / T + \phi) . \tag{12}$$

Here

$$\omega_L \equiv eb(1+a) / mc\gamma , \tag{13}$$

while $2\pi / T \equiv \omega_C$, the muon revolution frequency, so $2\pi kt / T \equiv k\omega_C t$.

Consider first the case $\phi = 0$. In this case, equations (10)-(12) have a simple analytical solution from which we can see that the perturbation $B_L(t)$ with the same frequency and phase as $\omega_a(t)$, see (6), (7) is not dangerous. To get the solution we can simply introduce a new variable u, $du = dt \cdot \cos 2\pi kt / T$, after which we have equations with constant coefficients. I will show here only the solution for $s_V(t)$, our main concern. If the initial time $t_0 = 0$, and the initial spin projections $s'_V(0) = s'_{V0}$, $s'_R(0) = s'_{R0}$, then for $\phi = 0$

$$s'_V(t) = s'_{V0} - s'_{R0} \frac{\omega_L}{k\omega_C} \left[1 - \frac{\delta\omega_a^2 + \omega_L^2}{6(k\omega_C)^2} \sin^2 k\omega_C t \right] \sin k\omega_C t . \tag{14}$$

In (14), there are only small fast oscillations of the vertical spin around its initial value.

The situation is radically different when $\phi \neq 0$. The out-of-phase mode of B_L, $b \sin\phi \cdot \sin k\omega_C t$ in this case, since we define the ω_a-oscillations as $\omega_a = \delta\omega_a \cos k\omega_C t$, rotates spin in the vertical plane. Let us consider a case with only this mode. The equations now are:

$$ds'_L / dt = s'_R \delta\omega_a \cos k\omega_C t \tag{15}$$

$$ds'_R / dt = -s'_L \delta\omega_a \cos k\omega_C t - s'_V \omega_L \sin\phi \sin k\omega_C t \tag{16}$$

$$ds'_V / dt = s'_R \omega_L \sin\phi \sin k\omega_C t \tag{17}$$

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There are no variables now that can help to simplify the equations. The solution of (15)-(17) can be found, however, as an infinite Fourier series. The preliminary analysis had shown that this series is a sum of terms with a low frequency connecting only the longitudinal, s'_L, and the vertical, s'_V, projections, and terms with high frequencies of the orders $k\omega_C$, $2k\omega_C$, etc. Thus, assume

$$s'_L(t) = s'_{L,slow}(t) + s'_{L,fast}(t); \quad s'_V(t) = s'_{V,slow}(t) + s'_{V,fast}(t). \tag{18}$$

In the first approximation, we can neglect the time derivatives of the slow functions. This immediately gives us the first approximation for s'_R, which is a fast function:

$$s'_R(t) = -\frac{\delta\omega_a}{k\omega_C} s'_{L,slow} \sin k\omega_C t + \frac{\omega_L}{k\omega_C} s'_{V,slow} \sin\phi \cos k\omega_C t. \tag{19}$$

Substituting this into equations (15) and (17), and keeping there only slow functions (in this first approximation), we get equations connecting the vertical spin projection with the longitudinal one.

$$ds'_{L,slow}/dt = \frac{\omega_L \delta\omega_a}{2k\omega_C} \sin\phi \cdot s'_{V,slow} \tag{20}$$

$$ds'_{V,slow}/dt = -\frac{\omega_L \delta\omega_a}{2k\omega_C} \sin\phi \cdot s'_{L,slow}. \tag{21}$$

Therefore, the muon spin in this case is rotated in the vertical plane with the angular frequency

$$\Omega_V = \left|\frac{\omega_L \delta\omega_a \sin\phi}{2k\omega_C}\right|. \ll \frac{\eta a}{2}\beta\gamma\omega_C \tag{22}$$

It is amazing that the spin rotates around the radial, not the longitudinal axis—as a result of the combination of the "oscillating rotations" around the longitudinal axis (perturbation ω_L), and the "oscillating rotations" around the vertical axis (perturbation $\delta\omega_a$).

The next approximations are not important. The technique of separating slow and fast oscillations that I have used here is a well-known one; it was apparently first developed by Lyapunov (in the middle of the 19th century) and then by Bogolyubov and others (in the middle of the 20th century).

$$\langle B_L(s)\,\delta\omega_a(s)\rangle \neq 0$$

Result:

Spin rotation not around L-axis (due to B_L), or around V-axis (due to $\delta\omega_a$), but around R-axis!

$$\Omega = \left|\frac{\omega_L\,\delta\omega_0 \sin\phi}{2k\omega_c}\right|$$

We need

$$\boxed{\left|\frac{\omega_L\,\delta\omega_{a0} \sin\phi}{2k\omega_c}\right| \ll \frac{\eta}{2}\beta\gamma\omega_c}$$

What is going on?

Why around the R-axis?

$$\langle B_L(s)\, \delta\omega_a(s) \rangle \neq 0$$

Why the case $\cos\phi = 1$, $\sin\phi = 0$

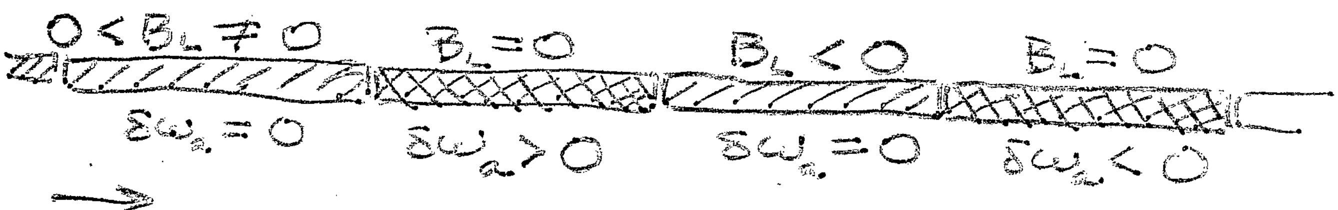
is different?

$B_L > 0$	$B_L = 0$	$B_L < 0$	$B_L = 0$	$B_L > 0$
$\delta\omega_a > 0$	$\delta\omega_a = 0$	$\delta\omega_a < 0$	$\delta\omega_a = 0$	$\delta\omega_a > 0$

Two simultaneous rotation, and then inverse of this operation — COMMUTE.

$$\langle B_L(s)\,\delta\omega_a(s)\rangle \neq 0$$

The answer: non-commutativity of rotations.



$\sin\phi = L.$

$$(\Phi_V^{-1})(\Phi_L^{-1})\,\Phi_V\,\Phi_L\begin{pmatrix}\text{Spin}\\\text{state}\end{pmatrix} \neq \begin{pmatrix}\text{The same}\\\text{state}\end{pmatrix}$$

$$\left[\Phi_V^{-1}\Phi_L^{-1}\right]^{-1} = \Phi_L\,\Phi_V \neq \Phi_V\,\Phi_L$$

$$\Phi_V\,\Phi_L\begin{pmatrix}\text{Spin}\\\text{state}\end{pmatrix} \neq \Phi_L\,\Phi_V\begin{pmatrix}\text{The same}\\\text{state}\end{pmatrix} =$$

$$= \left(\Phi_V\,\Phi_L + \xi\,\Phi_R\right)\begin{pmatrix}\text{Spin}\\\text{state}\end{pmatrix}$$

So spin is rotating around the R-axis

Estimations

$$d_d = \eta_d \times 2.7\times 10^{-15}\ e\text{-cm}$$

We need to measure up to

$$\eta_d \sim 4\times 10^{-14}$$

$$(\eta_d)_{false} = \left(\frac{\delta\omega_a}{\omega_c}\right)\vartheta_L \frac{\sin\phi}{k\beta\gamma} \quad \text{for some } k$$

$$\sqrt{(\eta_d^2)_{false}} = \left(\frac{\delta\omega_a}{\omega_c}\right)\vartheta_L \frac{1}{2\beta\gamma}\left(1+\frac{1}{2^2}+\dots+\frac{1}{32^2}\right)^{1/2} =$$

$$= \left(\frac{\delta\omega_a}{\omega_c}\right)\vartheta_L \frac{\sqrt{1.54}}{2\beta\gamma} =$$

$$= 1.38\left(\frac{\delta\omega_a}{\omega_c}\right)\vartheta_L \qquad (\text{for } E_R = 3.5\ \text{MeV}/c)$$

Thus, $\vartheta_L(\delta\omega_a/\omega_c) \ll 2.0\times 10^{-14}$

$$\sqrt{\overline{\vartheta_L^2}}\ \sqrt{\overline{\delta\omega_a^2}/\omega_c^2} \ll 0.70\times 10^{-12} \quad (\text{for } N=32)$$

Thus,

$$\sqrt{\overline{\vartheta_L^2}} = 10^{-6} \text{ and } \sqrt{\overline{\delta\omega_a^2}/\omega_c^2} = 10^{-6}$$

will be sufficient.